Existence and Uniqueness of Solutions in Nonlinear Differential Equations

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Abstract: The existence and uniqueness solutions of the following initial value problem (I.V.P.)

$$x'(t) + a\,x(t) = f(t, x(t)) + r(t),\ \ x(0) = x_0,\ \ t \geq 0,$$

is studied in this paper. First, this I.V.P. is inverted to an equivalent integral equation. The existence of solutions is then obtained from the equivalent integral equation. Banach Space, Schauder's fixed point theorem, Gronwall's Inequality and the contraction mapping principle are used in the analysis.

Under certain conditions in f, Schauder's Fixed Point Theorem is used to obtain the existence of at least one solution of this I.V.P. on $[0, \infty)$. Then, employing Gronwall's Inequality, it is shown that if f is globally Lipschitz, then the I.V.P. has indeed a unique solution on $[0, \infty)$. Finally, the existence of a unique solution of this I.V.P. is obtained by employing the Contraction Mapping Principle, however; a restrictive condition is shown to be required in this method. This restriction will not be required in the method that involves Schauder's Fixed Point Theorem and Gronwall's Inequality.