Finding the Grundy Number of line graphs

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Abstract: A Grundy edge-coloring of a graph is a proper (adjacent edges get different colors) assignment of positive integers to the edges of the graph such that if an edge is colored (assigned) $c > 1$, then the edge is adjacent to edges of all the colors $1, \ldots, c$. The edge-Grundy number of a graph $G$ (denoted $\Gamma'(G)$) is the largest positive integer appearing on an edge of the graph, among all the colors appearing on all the Grundy edge-colorings of the graph. The practical significance of this number is its function as an index of a worst possible outcome during online or greedy proper colorings of the graph; such colorings are useful in scheduling. In this work we determine the edge-Grundy numbers of $G$ in a number of cases including the complete graphs, the complete bipartite graphs, the hypercubes, the grids, the regular complete multi-partite graphs, and give bounds on edge-Grundy numbers of some non-regular complete multi-partite graphs.