A Stone type Duality between profinite MV-algebras and multisets

Speaker: JEAN B NGANOU

Date: February 20, 2014

MV-algebras were introduced by C. Chang (1958) as the algebraic counterpart of Łukasiewicz many-value logic. MV-algebras play for the Łukasiewicz many-value logic the same role that Boolean algebras play for the two-value logic. Undoubtably, one of the most important theorem on MV-algebras due to Mundici is the equivalence between MV-algebras and Abelian lattice-ordered groups with strong units.
The talk will start with the basic definitions, important examples of MV-algebras, and a brief description of the equivalence cited above. Profinite algebras are very important objects and understanding these in various categories has proven to be crucial in several important settings. Most likely, the best known examples are in number theory, where the ring of p-adic integers can be constructed as the inverse limit of the finite rings $\mathbb{Z}/p^n\mathbb{Z}$. An equally popular example comes from the theory of infinite Galois theory, where the Galois group of infinite Galois extensions can be expressed as inverse limits of finite Galois groups. The main focus of the talk will be presenting my recent results about profinite MV-algebras and their equivalence (of Stone type) to multisets. First, we shall completely characterize the profinite MV-algebras and obtained that they are eaxctly products of finite Łukasiewicz chains. Multisets are defined in combinatorics as pairs $\langle X, \sigma \rangle$, where X is a set and $\sigma : X \to \mathbb{N}$ assigning to each x its multiplicity $\sigma(x)$. The main result is that the category $\mathbb{M}$ of multisets is dually equivalent to the category $\mathbb{P}$ of profinite MV-algebras and homomorphisms that reflect principal maximal ideals.
I will end the talk by describing my ongoing joint work with E. Marra on profinitely presented MV-algebras, a significantly more complex class of algebras than profinite ones.