

An Inverse Semi-Group Approach to Classification of Cartan Pairs

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ABSTRACT: In a pair of landmark papers appearing in 1977, Feldman and Moore studied Borel equivalence relations on a measure space (X, μ) and developed a cohomology theory for such relations. They used their work to classify Cartan pairs, which are pairs (M, D) consisting of a von Neumann algebra M with a suitable maximal abelian subalgebra D . The remarkable feature of this work is that the inclusion of D in M can be used to introduce ‘coordinates’ on M , so that M can be thought of as an algebra of (generalized) matrices with (generalized) matrix operations analogous to the matrix operations on $n \times n$ matrices. This viewpoint allows considerable insight into the structure of M .

The Feldman-Moore work requires considerable measure-theoretic prowess, and the use of equivalence relations for classification is more “point-based” than “operator based.”

In this talk, I will discuss a new classification of Cartan pairs due to Donsig, Fuller and Pitts, in terms of extensions of inverse semigroups. This classification is algebraic in nature, suppresses the measure theory, and seems conceptually simpler than the Feldman-Moore approach. If time permits, I will show how our approach can be used to give a correct proof of an interesting assertion of Muhly, Saito and Solel regarding maximal triangular algebras.