The Mathematics of Arctic Tipping Points

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Abstract: Important elements of the Arctic climate system such as sea ice and permafrost were formed under temperature change resulting in phase transition, primarily between water and ice. Phase transitions in the Arctic cryosphere remain of continuing interest as the climate system warms, and are crucial for the stability of the climate system. Also, such phase transitions lead to changes in shapes of the different patterns in the Arctic Cryosphere.

During the Arctic melt season, melt pond (that develops on the surface of sea ice floes) geometry has a complex fractal structure as a result of the fractal dimension transition. I will discuss a standard conceptual energy balance climate model with ice–albedo feedback taking into account the albedo of melt ponds. Different tipping points were obtained for this model. One of them may be quite interesting for climate projections, where the temperature of this system is stabilized (after phase transition) only due to the fractal transition in melt pond geometry.

Another important problem, where geometrical changes of the patterns can lead to the tipping points in a climate system, is modeling permafrost lakes. I will discuss the nonlinear phase transition model for permafrost which was used to compute the methane emission generated by lakes in the Arctic. Simulations show that there are two different possible scenarios for methane emission. One of them lead to abrupt temperature change, that is a tipping point in the climate system.