PRICING THE ASIAN CALL OPTION

VINH XUAN DANG,
SCOTT GLASGOW,
HARRISON POTTER,
STEPHEN TAYLOR

ABSTRACT. Background material on measure-theoretic probability theory and stochastic calculus is provided in order to clarify notation and inform the reader unfamiliar with these concepts. These fields are then employed in exploring two distinct but related approaches to fair option pricing: developing a partial differential equation whose solution, given specified boundary conditions, is the desired fair option price and evaluating a risk-neutral conditional expectation whose value is the fair option price. Both approaches are illustrated by example before being applied to the Asian call option.

Two results are obtained by applying the latter option pricing approach to the Asian call option. The price of an Asian call option is shown to be equal to an integral of an unknown joint distribution function. This exact formula is then made approximate by allowing one of the random variables to become a parameter of the system. This modified Asian call option is then priced explicitly, leading to a formula that is strikingly similar to the Black-Scholes-Merton formula, which prices the European call option. Finally, possible methods of generalizing the procedure to price the Asian call option both exactly and explicitly are speculated.

I. INTRODUCTION: FINANCIAL MOTIVATION

An understanding of the financial issues that give the mathematics problems presented here meaning is critical to following the overall reasoning. With this end in mind, a brief overview of relevant financial background material is provided.

There are essentially three forms that wealth can take in financial models: money in the money market, shares of an asset in the stock market, and stakes in an option. Any wealth in the money market will grow in accordance with a given interest rate. Wealth invested in the money market is not expected to have a particularly impressive growth rate, but it is considered to be a reliable investment as it will generally steadily increase in value. On the other hand, investing directly in an asset is an inherently risky endeavor as its value will fluctuate both up and down in a random manner; however, the potential for loss is counterbalanced by the potential for greater gain. This is what attracts casual investors with dreams of instantly becoming wealthy to bet on the stock market; however, this is not the way that most major financial institutions invest in assets.

From the perspective of a casual investor, purchasing an option from a financial institution is risky in much the same way that investing in an asset is risky: the final value of the option is dependent upon the asset’s values over the duration of the contract, and there is thus again the potential for loss. It simply changes the way in which the risk is managed. But this, in fact, is the real value of an option: by enabling the reallocation of risk, options serve a purpose that is the financial world’s analog of insurance. A business, for example, might buy an option that will increase in value if its competitors do well, but will decrease