A New Spin on Baseball

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All baseball fans know what a curveball is physically, but what is curveball mathematically, and how does it differ from a fastball? The secret of a pitch lies in its spin. In this paper we shall define the spin of a baseball and investigate the effects of its magnitude and direction by employing data collected by MLB.com Gameday from the league's best pitchers. We shall then employ this model to differentiate between the spin of a curveball and that of a fastball.

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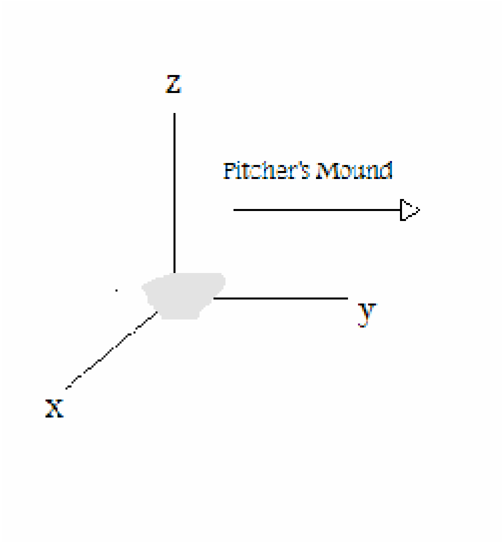
The pitcher winds and delivers a perfectly thrown curveball, freezing up the batter. After being completely humiliated the batter walks back to the dugout wondering what causes a baseball to move that way. The answer is spin. We shall investigate how spin affects the acceleration of a baseball pitch. Using data from the Gameday feature on MLB.com [2], we shall demonstrate the difference between a fastball and a curveball.

But what is spin? As employed here, spin is defined as a vector that points along the axis of rotation. The magnitude of this spin vector is the number of revolutions per second of the baseball. In physics, this is known as an angular velocity vector. An object of any shape rotating around a fixed axis of rotation has an angular velocity vector. For our purposes we shall be dealing with a baseball with an axis of rotation fixed relative to the baseball through its center of mass. The spin vector shall be orthogonal to the velocity vector in order to simplify our examination of the pitches. But why is the spin important? In defining a fastball and how it differs from a curveball, the spin is vital. The spin of a pitched ball causes differences in air pressure around the ball, which cause the ball to accelerate, so along with gravity, the spin of a pitched baseball determines the trajectory of the pitch. Before we can go on, we must establish a coordinate system.

The coordinate system that appears most suitable places the positive x direction to the first base side of home plate, the positive y direction towards the pitcher's mound from home plate, and, the positive z direction straight up from the catcher. Since all the pitches travel from the pitcher to the catcher, they move in the negative y direction.



As a pitcher throws a fastball, the ball is rolled off the fingers with a backwards spin. The right hand rule convention places this spin vector in the negative x direction. The same approach is taken in assigning a spin vector to the curveball. When the pitcher releases the curveball, he is essentially "flicking" it off his finger in a way that creates forward spin. The right hand rule works in the same manner, except now the spin vector is in the positive x direction. Furthermore, the magnitudes of the spin vectors corresponding to the fastball and the curveball vary. We employ a model from Edwards + Penny Calculus 6e [1], and insert data from the Gameday feature at MLB.com [2], which we shall discuss later, to find the differences in magnitude.

The given Edwards and Penny model [1] shows the acceleration of the ball based on the spin vector, the initial velocity vector, and the acceleration due to gravity. This model is further verified in the studies of aerodynamics, which say that spin causes differences in air pressure on both sides of the ball, thus creating a spin acceleration as shown:

$$\mathbf{a} = (c\mathbf{S} \times \mathbf{v}_0) - g\mathbf{k} \qquad (1)$$

In this model $\mathbf{a}$ represents the acceleration of the baseball in the x, y, z coordinate system; $\mathbf{S}$ denotes the spin vector of the baseball; $\mathbf{v}_0$ symbolizes the initial velocity of the baseball in the x, y, z coordinate system; $g\mathbf{k}$ signifies the baseball's acceleration due to gravity, equal to 32 feet per second squared in the z negative direction; and c indicates an empirical constant that was given in the model as .005.

In order to compare the spin of a curveball and that of a fastball, we have to algebraically solve for the spin vector, $\mathbf{S}$, from equation (1). Since the accelerations with which we are working already take gravity into account, and we want to find just the acceleration due to spin, we add the gravitational acceleration, $g\mathbf{k}$, to the z component of the original acceleration. Next we divide each term of the acceleration by the empirical constant c. We then call this modified acceleration $\mathbf{a}_m$.

$$\mathbf{a}_m = \left\langle \frac{\mathbf{a_x}}{\mathbf{c}}, \frac{\mathbf{a_y}}{\mathbf{c}}, \frac{\mathbf{a_z} + g}{\mathbf{c}} \right\rangle \tag{2}$$

So now:

$$|\mathbf{a}_m| = |\mathbf{S} \times \mathbf{v}_0| \tag{3}$$

After isolating $\mathbf{S} \times \mathbf{v}_0$ we need to determine the magnitude of $\mathbf{S}$. This magnitude $|\mathbf{S}|$ denotes the number of revolutions per second of the baseball.

One theorem of cross products states:

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta \tag{4}$$

Using this theorem we derive a customized equation for our variables:

$$|\mathbf{a}_m| = |\mathbf{S}||\mathbf{v}_0|\sin\theta \tag{5}$$

We now solve for the magnitude of the spin $|\mathbf{S}|$:

$$|\mathbf{S}| = \frac{|\mathbf{a}_m|}{|\mathbf{v}_0|\sin\theta} \qquad (6)$$

The data with which we are working give us no value for θ, the angle between the spin and initial velocity vectors. We have to decide on some value of θ that we can keep constant throughout our calculations for both pitches to determine a significant spin value. Since we have values for $\mathbf{a}_m$ and $\mathbf{v}_0$, making θ 90 degrees maximizes the denominator ($\sin 90 = 1$) and therefore minimizes the spin value.

Through employment of the defined model and extracted data from MLB.com's Gameday application, a fastball and a curveball can now be sensibly compared. Gameday is an online feature that tracks and records live data on every pitch in any Major League Baseball game. This data includes the position, velocity, and acceleration of the pitches as well as a variety of other information. Only some of the data applies to our research, so we employ only the needed information.

What in the data distinguishes a fastball from a curveball? A curveball always has an acceleration magnitude in the z direction greater than the acceleration due to gravity alone because of the direction of the spin causes the baseball to accelerate downward. A fastball always has an acceleration magnitude in the z direction less than the acceleration due to gravity because of the spin direction, which causes the baseball to accelerate in the upward direction. Another determining factor is the initial speed of the pitches. A typical curveball has a speed under 80 mph, whereas a fastball is conventionally at speeds in excess of 87 mph.

The selected pitchers are Bronson Arroyo of the Cincinnati Reds for his curveball and Josh Beckett of the Boston Red Sox for his fastball. Here is some of the collected data:

For one curveball:

$$\mathbf{a} = < 4.384, 19.498, -36.315 >$$
$$\mathbf{v}_0 = < 5.295, -98.446, -1.211 >$$

We then find the modified acceleration by using the process previously described:

$$\mathbf{a_m} = < 876.8, 3899.6, -863.0 >$$

Next we find the magnitude of the velocity, $|\mathbf{v_0}|$, and the magnitude of the modified acceleration, $|\mathbf{a_m}|$.

$$|\mathbf{v_0}| = 98.595732\text{ft/sec}$$

$$|\mathbf{a_m}| = 4089.0619\text{ft/sec}^2$$

Inserting these values into the customized equation yields the minimum spin magnitude for this pitch:

$$\min|\mathbf{S}| = \frac{4089.0619}{98.5957\sin 90} = 41.4730\text{rev/sec}$$

By evaluating other pitches via the same process these minimum spin magnitudes are obtained:

	Min $\lvert\mathbf{S}\rvert$ (rev/sec)
1	41.473011
2	46.179763
3	50.823846
4	44.156804
5	45.669365
6	42.458440
7	47.258479
8	50.928479
9	45.373656
10	49.326510
AVG.	46.364806

The same procedure is then employed for Josh Beckett's fastballs producing these minimum spins:

	Min $|\mathbf{S}|$ (rev/sec)
1	69.299733
2	68.061145
3	64.815963
4	63.128792
5	62.859936
6	67.919915
7	60.606146
8	60.791572
9	67.507436
10	68.120090
AVG.	65.311073

Although the motion of a curveball suggests more spin, this is in fact untrue. Through these findings it is evident that on average a fastball actually spins more. The final average values of the spin magnitudes are:

Fastball: 65.311073 rev/sec

Curveball: 46.364806 rev/sec

Difference: 18.946267 rev/sec

This difference can be explained because a fastball opposes gravity whereas a curveball does not oppose gravity. The spin of a fastball causes acceleration in the positive z direction. Although not enough to completely cancel out the acceleration of the ball due to gravity, it does cause the ball to have a relatively straight trajectory. The greater the spin magnitude, the less the pitch will deviate from a truly straight trajectory. Conversely, the spin of a curveball causes acceleration in the negative z direction. This adds to the acceleration caused by gravity. Therefore a curveball has a much greater deviation from a straight trajectory than does a fastball. Thus, the batter's question is answered. Next time he will know to more closely watch the spin of the pitch if he wishes to get a hit.

References

[1] Edwards, C. Henry and David E. Penney. Calculus 6e. New Jersey: Prentice Hall, 2002.

[2] "Index of/components/game/mlb." Major League Baseball. 10 Oct.2007 <http://gd2.mlb.com/components/game/mlb>.