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HOW ONE'S RISK PREFERENCES AFFECT THEIR INVESTMENT DECISIONS

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Abstract: The purpose of our project was to display how our personal risk preferences affect our investment decisions, if we invested on two assets: one risky asset (stock) and one risk-free asset (bank account). We considered the problem in both discrete and continuous case. In particular, the stock price follows a multinomial tree in the discrete case; and follows a Geometric Brownian motion in the continuous case. We then found the expected value of the stocks at varying times. By setting what we expect our bank account to be at those times equal to these expected values, we solved for the interest rates, at which investing on either asset are equivalent. We then incorporated risk aversion in the power utility function. Using different levels of risk aversion, we again solve for the interest rate, at which investing on either asset are equivalent. By comparing the first interest rate with the interest rate that incorporated the risk aversion, we saw how this risk aversion affects our investment decisions.

1. INTRODUCTION AND BACKGROUND

To first understand this presentation you need to first have a basic understanding of consumer, utility, and prospect theory. Consumer theory hopes to understand the behavior of consumers and in doing so predict what they may do next. When analyzing consumer theory, we analyze bundles of goods, and these bundles have certain assumptions. We assume completeness, which is, that given the two bundles; we will either prefer one to the other or be completely indifferent. We also assume transitivity, meaning that if we prefer a first bundle to a second, and the second to a third, then we will prefer that first bundle to the third. Finally, we also assume monotonicity, meaning that more of a good will always be better. From this consumer theory, we get the theory of utility. Utility allows us to numerically represent the bundle preferences. A utility function gives actual values to certain consumption bundles, the higher the utility the higher the happiness.

Expected utility then just incorporates the probabilities of certain occurrences, with the utility that they give consumers. So, the expected value will be the sum of the probabilities of certain occurrences, multiplied by the utility they give to consumers. Thus, this is not telling you how much money is expected to be made, rather the happiness that is expected to be experienced from this set of choices. This can sometimes result in a different choice being the better one. From these utilities, one can create utility functions. These functions describe the amount of utility had based on another factor, often wealth. Based on before stated assumptions about bundles and utility there are certain factors shared by utility functions. Firstly, based on monotonicity, utility function will be increasing as more of a good will always produce a higher utility. Secondly, it is not hard to see that these utility functions will be subject to diminishing marginal returns. When you have a large amount of wealth, the effect of increasing it by one dollar will be much less than the effect of increasing it by this same amount when you have a small amount of wealth. Understanding the reason behind this is not difficult. Thus, most utility functions will be concave down.

When we get into prospect theory, though we begin to see that some of these assumptions do not hold. Take a portion of the Allais experiment [1]. Consider a first set of choices in which you will receive money,

A: $5,000 with probability .1
$1,000 with probability .89
0 with probability .01

B: $1,000 with probability 1

And then a second set of choices

C: $5,000 with probability .1
0 with a probability .9

D: $1,000 with probability .11
0 with probability .89

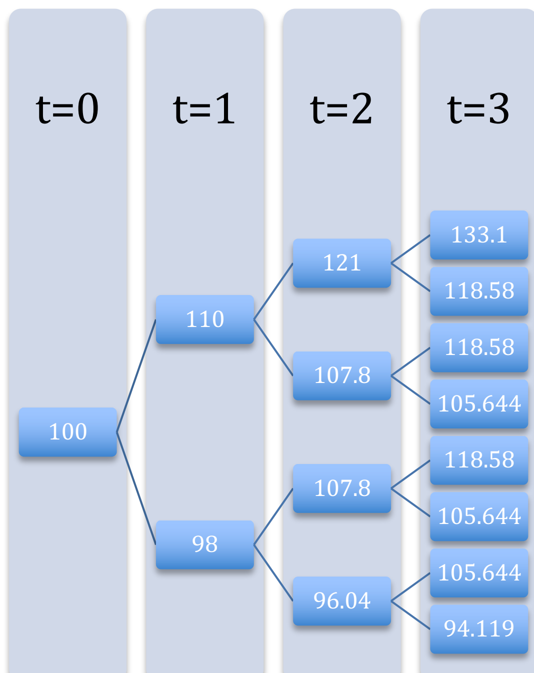
in the first set of choices most would choose choice B, while in the second set of choices most would choose choice C. Yet, according to the utility assumptions if you prefer B to A, then you should prefer D to C, yet this is not the case. This is where risk aversion, which has not yet been included in the utility model, is also affecting a consumer's decision. Because of risk aversion in the first set of choices a consumer underweights the probability of .89 because they are risk averse enough that they wish to simply choose the risk-free option. Then in the second decisions, when the consumer is forced to choose a risky option, this underweighting does not occur, thus a different choice is made. Thus, to better understand consumers behavior we used the Constant Relative Risk Aversion (CRRA) utility function. This function describes utility of consumers, while incorporating risk aversion. It does this while still following the standards of utility functions of monotonicity and diminishing marginal utility. The function used is

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

in which γ represents the risk aversion.

2. EXPERIMENT

We displayed these effects using both a discrete and continuous case. In both cases we compared the decision to invest in a bank account (a risk-free asset,) or a stock (a risk asset). We will first consider the discrete case. For this case, we assumed an initial wealth, x, of $100. We then assumed that the stock followed a multinomial tree in which it either increased from the initial wealth by 10% or decreased by 2%, each with a 50% chance of occurrence. This was then carried out for three time periods.



This diagram represents the behavior of the stock.
The bank account was then represented by the equation

$$B_t = 100(1+r)^t$$

In this equation r represents the interest rate.

We wished to find the interest rate at which investing in the stock would be equal to investing in the bank at specific times. We did this by setting the expected value of the stock ($E[S_t]$) at certain times equal to the expected value of the bank account ($E[B_t]$) at those same times. To find the expected value of the stock we simply multiplied each possible return from the stock by its corresponding probability and added these. We can treat the entire bank account equation, as a constant thus the expected value will simply be the same equation. We then did this process and solved for r at times [0,1,2,3]. The interest rate found was the same at all times, it was a rate of 4%.

We then wished to incorporate risk aversion and did so by using the CRRA utility function mentioned above.

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

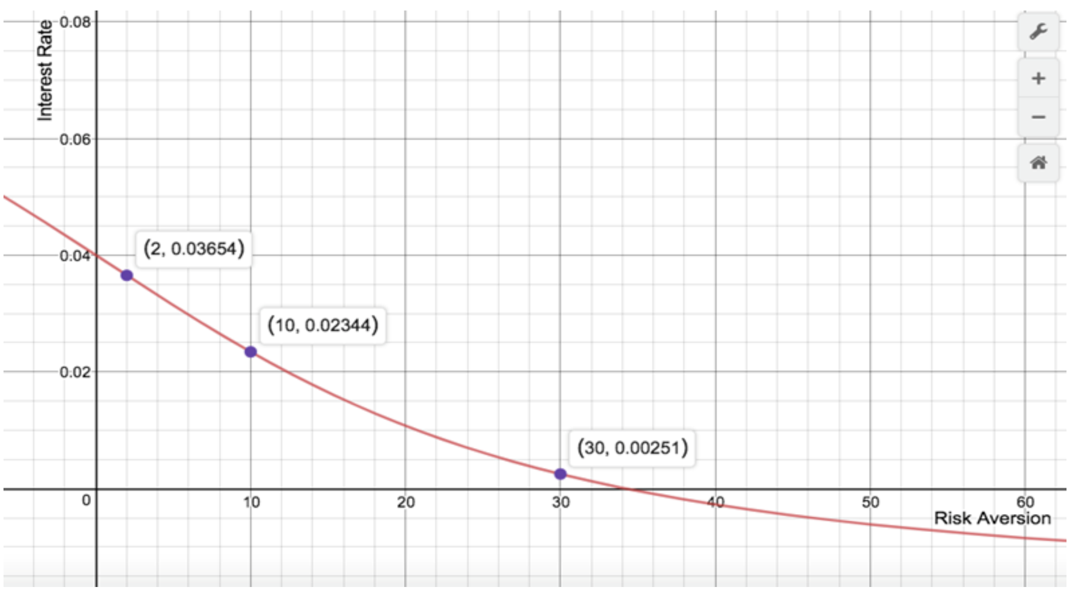
We now followed the same process, yet this time we instead used the expected value of the utility of the stock ($E[U(S_t)]$) and the expected value of the utility of the bank account ($E[U(B_t)]$). We did this for times t=(1,2,3) and risk aversions of $\gamma =$ (2,10,30). The results can be seen in the following charts

Risk Aversion (γ) = 2	
Time (t)	Interest Rate (r)
1	3.653846154
2	3.653846154
3	3.653846154

Risk Aversion (γ) = 10	
Time (t)	Interest Rate (r)
1	2.344416758
2	2.344416758
3	2.344416758

Risk Aversion (γ) = 30	
Time (t)	Interest Rate (r)
1	0.251285757
2	0.251285757
3	0.251285757

As can be easily seen as time changes it does not affect the interest rate thus to see the effect of interest rate as the risk aversion changes we can choose to show it at any of the three times. We chose to show this at time one. The results can be seen the following graph



We then considered the continuous case, using a similar process as we did in the discrete case. For this case, we again assumed an initial wealth, x, of $100. We then assumed that the stock price at time t and the bank account were represented by the following equations:

$$S_t = xe^{\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}Z\}} \qquad Z \sim N(0,1)$$

$$B_t = xe^{rt}$$

In these equations, we supposed that μ = 8% and σ = 20% and r represents the interest rate.

We wished to find the interest rate at which investing in the stock would be equal to investing in the bank. We did this by setting the expected value of the stock equal to the expected value of the bank account at those same times. We then did this process and solved for r at times [0,1,2,3]. The interest rate found was the same at all times, a rate of 8%.

We then wished to incorporate risk aversion and did so by using the CRRA utility function mentioned above. We now followed the same process, yet this time we used the expected value of the utility of the stock and the expected value of the utility of the bank account, which can be represented by the following equations:

$$E[U(B_t)] = \frac{(xe^{rt})^{1-\gamma}}{1-\gamma} \text{ and,}$$

$$E[U(S_t)] = \frac{x^{1-\gamma} e^{t(1-\gamma)(\mu - \frac{\sigma^2}{2}\gamma)}}{1-\gamma}$$

When setting $E[U(B_t)] = E[U(S_t)]$ and solving for r, we found the following equation:

$$r = \mu - \frac{\sigma^2}{2}\gamma = .08 + .02\gamma$$

We found the interest rate (r) for times t= (1,2,3) and risk aversions of 2, 5, and 10. The results can be seen in the following chart

Time = 1,2, and 3	
Risk Aversion (γ)	Interest Rate (r)
2	0.04
5	-0.02
10	-0.12

As can be seen, as time changes it does not affect the interest rate. Therefore, to see the effect of interest rate as the risk aversion changes, we can choose to graph it at any of the three times. The results can be seen in the following graph:

3. CONCLUSION

As we have discussed in the introductory paragraphs risk aversion is something that nearly every human being has. And from the above results it is not difficult to see how. For both the discrete and continuous cases the breakeven interest rates that incorporated risk was much lower than that did not. To continue on this point, as the risk aversion grew so did the breakeven interest rate. When we say breakeven interest rate, we are referring to the interest rate at which it is of the same expected utility to invest your money in the stock market, as it is to invest your money in the bank. Thus, it makes sense that this would get lower, as you become more and more risk averse, you will be willing to accept a lower and lower interest rate from the bank, thus a lower return on your money, in return for the avoidance of the risk that would go along with investing in the stock market. At certain levels of risk aversion, it could be seen that the interest rate even became negative. This means, that at these levels of risk aversion, an investor would be willing to sacrifice a known amount of his wealth, or basically pay, to avoid the risk of losing all of his wealth in the stock market. These above analysis show that if one wishes to analyze investors behavior, they cannot simply find where the two equations alone would be equal, but they must incorporate both an investor’s utility and their risk aversion to get a clear understanding of what they might do and why.

REFERENCES

1. Allais, Maurice, *Le Comportement de l\'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l\'Ecole Americaine*, Econometrica, (1953)

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