

THE EFFECT OF THERMAL MASS ON THERMAL TRANSMISSION LOADS

Robert B. Gilbert MME, Kelly Kissock Ph.D, P.E.

Department of Mechanical and Aerospace Engineering
University of Dayton
Dayton, Ohio USA

ABSTRACT

A finite-difference model is used to simulate the effects of thermal mass on thermal load. A sinusoidal function is used to simulate the exterior air temperature. The interior air temperature set point remains constant. The mechanism by which thermal mass affects thermal load is described. Equations are given to calculate thermal load as a function of the exterior air temperature amplitude, the temperature above and below the mean temperature. Equations are given to calculate the minimum thermal load resulting from thermal mass and therefore the maximum thermal load reduction with thermal mass. Equations and methods are given to calculate the minimum thermal load resulting from thermal mass from weather data files, TMY or TMY2 files. A design equation is given as a guideline to determine the amount of thermal mass required to reduce the thermal load with given weather conditions.

INTRODUCTION

Thermal mass is known to reduce thermal building loads. For example, the BESTEST protocol describes cases where the thermal load was reduced by more than 60 % in a heavyweight building compared to a lightweight building [1]. However, the mechanism by which thermal mass affects thermal loads is not widely understood or easily quantified. Moreover, different weather conditions result in different thermal load reductions with increasing thermal mass. The purpose of this study is to make explicit the mechanism by which thermal mass affects thermal transmission loads and to offer design equations to determine the maximum thermal load reduction which can be obtained with thermal mass. The purpose is also to offer design approximation equations to determine the amount of thermal load reduction as a function of thermal mass as a guide to the amount of thermal mass required.

Gajda, Marceau and VanGeem [2] demonstrate temperature damping and time lag with thermal mass, but the mechanism by which thermal load is reduced and under what specific conditions thermal load is reduced are not defined. Kalogirou, Florides, and Tassou [3] conclude a south-facing thermal mass wall offers an advantage. The advantage of non-south-facing

thermal mass walls with cyclic exterior temperature is not addressed.

FINITE-DIFFERENCE MODEL

Figure 1 shows the wall configuration used in the finite-difference simulations. The node locations and the thickness of each material are shown. The density of the exterior and interior insulation was assumed to be negligible compared to the density of the thermal mass; thus, insulation nodes were placed on the surface. The thermal mass was modeled with eight nodes for greater accuracy. Prior to selecting 8 nodes, the number of nodes was varied until there was confidence that the number of nodes did not affect the results. The simulation employed a one-dimensional, forward, explicit solution technique. Stability was guaranteed by choosing a time step resulting in $Fo \ll 0.5$ and $Fo(1 + Bi) \ll 0.5$. To eliminate initial conditions, the model was run until solutions converged to at least four significant figures.

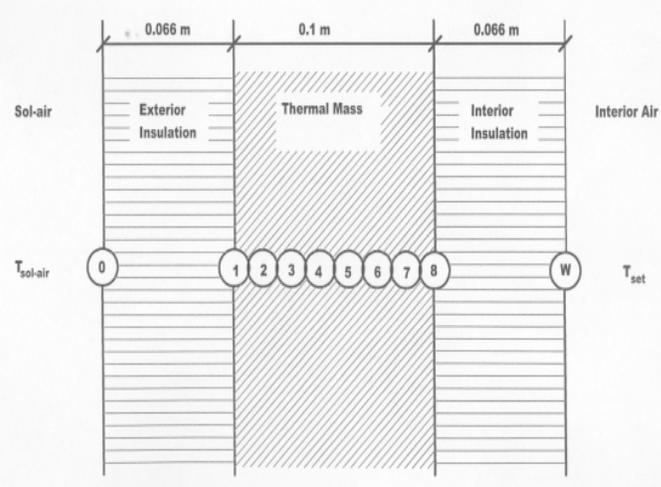


Fig. 1 Configuration used in finite difference simulation

Variable names, and values used in the simulation are shown in Table 1. These values correspond to a base wall consisting of 0.066 m of fiberglass batt insulation on the outside of 0.1 m of concrete. In the base case, the U value of the interior insulation was set to a high value so that the effect of the interior insulation would be negligible. In later runs, the U values of the exterior and interior insulation were reversed to simulate a wall with interior insulation. In addition, in later runs the density of the concrete was increased to simulate larger thermal mass.

Table 1 Variable names and values.

h_o	The exterior convection coefficient, $29.3 \text{ W/m}^2 \cdot \text{K}$
h_i	The exterior convection coefficient, $8.26 \text{ W/m}^2 \cdot \text{K}$
$T_{sol-air}$	The outside sol-air temperature which is simulated by a sinusoidal function, C° .
T_{set}	The interior air temperature set point, 20 C .
T_m	The average or mean outside sol-air temperature, C° .
$T_{mean sol-air}$	Also use as the average or mean outside sol-air temperature, C° .
$T_{mean node}$	The average or mean node temperature, C° .
T_{amp}	The absolute value of the temperature difference above and below the average, mean, temperature.
$T_{amp decay}$	The reduced of decayed absolute value of the temperature difference above and below the average, mean, temperature as a result of increased thermal mass.
$T_{amp sol-air}$	The absolute value of the temperature difference above and below the average, mean, sol-air temperature.
$T_{amp node}$	The absolute value of the temperature difference above and below the average, mean, node temperature.
Cp_m	The specific heat of the thermal mass, $J/kg \cdot K$. Its size varies.
Cp_{io}	The specific heat of the exterior insulation, $840 J/kg \cdot K$.
Cp_{ii}	The specific heat of the interior insulation, $840 J/kg \cdot K$.
ρ_m	The density of the thermal mass, kg/m^3 . Its value varied.
ρ_{io}	The density of the exterior insulation, $12 kg/m^3$
ρ_{ii}	The density of the interior insulation, $12 kg/m^3$
U_m	The thermal conductance of the thermal mass. Varied from $5.1 \text{ W/m}^2 \cdot K$ to $0.6375 \text{ W/m}^2 \cdot K$
U_{io}	The thermal conductance of the exterior insulation, $0.606 \text{ W/m}^2 \cdot K$ except when the U-value of the thermal mas was altered.
U_{ii}	The thermal conductance of the interior insulation, $79.5 \text{ W/m}^2 \cdot K$
tk_m	The thickness of the thermal mass, 0.1 m .
tk_{io}	The thickness of the exterior insulation, 0.066 m .
tk_{ii}	The thickness of the interior insulation, 0.066 m .

EFFECTS OF THERMAL MASS

The finite-difference model was used to complete a matrix of results of the effects thermal mass on thermal load.

The mean temperature of the outside sol-air temperature was varied from 0 C to 30 C. In each case, the amplitude of the outside sol-air temperature was varied for 0 C to the extreme of 40 C. Further, in each of the combinations of these cases, the thermal mass was varied from 0 to $1.4 \times 10^6 \text{ J/K} \cdot \text{m}^2$. No internal generation, infiltration, or windows were considered. The results of the effect of thermal mass on thermal load, with outside air sinusoidal temperature variation and constant inside air temperature, is presented with two mean temperature/amplitude temperature combinations. In both cases, the inside air temperature is fixed at 20 C and the outdoor air temperature varies with an amplitude of 10 C. In the first case, the mean exterior air temperature was 20 C, resulting in an outside air temperature varying from 10 C to 30 C. In the second case, the mean exterior air temperature was 15 C, resulting in an outside air temperature varying from 5 C to 25 C.

The first case, with a mean exterior air temperature of 20 C, demonstrates the effect of thermal mass on the time-temperature profile within the thermal mass. Figure 2 shows a time-temperature profile over a 24 hour cycle of three points within a hypothetical wall with zero thermal mass. The thermal mass was set to zero by setting the specific heat of the material equal to 0. Node 1 is on the outside surface of the thermal mass. Node 5 is in the mid portion of the thermal mass, and node 8 is on the inside surface of the thermal mass. The thermal mass has a thickness of 0.1 meter and a U-value of $5.1 \text{ W/m}^2 \cdot K$. In all three nodes, the temperature peaks at hour 15, which is when the exterior air temperature peaks. The temperature reduces linearly through the material as expected.

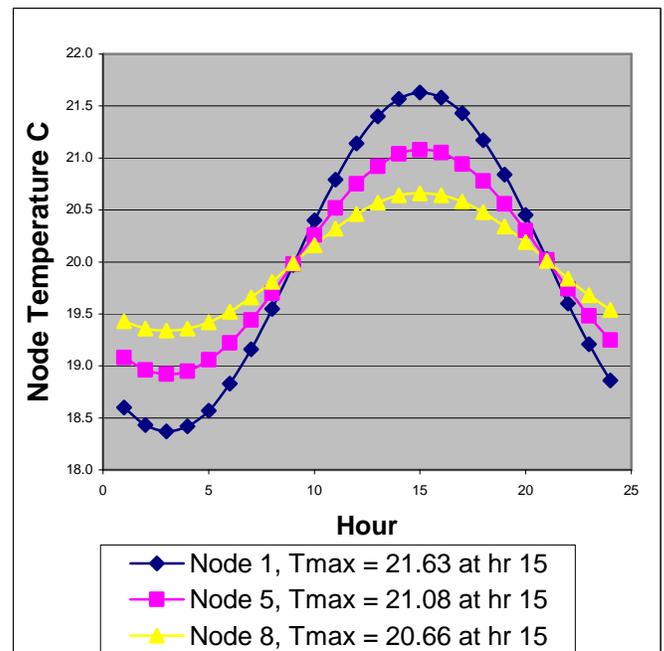


Fig. 2 24 Hour Temperature of Nodes 1, 5, and 8 with thermal mass = 0 J/km², $U_m = 5.1 \text{ W/m}^2 \text{K}$, $T_{set} = 20 \text{ C}$, $T_m = 20 \text{ C}$, $T_{amp} = 10 \text{ C}$

Figure 3 shows the same nodes as Figure 1, but the material has a thermal mass of $1.4 \times 10^5 \text{ J/K}\cdot\text{m}^2$. The peak temperature of node 1 is reduced from 21.63 C to 20.85 C, and the hour is now 18, a delay of 3 hours. The peak temperature of node 5 is reduced from 21.08 C to 20.53 C and the peak hour is 20, a delay of 5 hours. The peak temperature of node 8 is reduced from 20.66 C to 20.32 C and the peak hour is now greater than 20, a delay of greater than 5 hours. The reduction in temperature of node 8 is only 0.34 C but represents 52 % of the temperature difference between the inner surface of the thermal mass and the inside air temperature set point of 20 C.

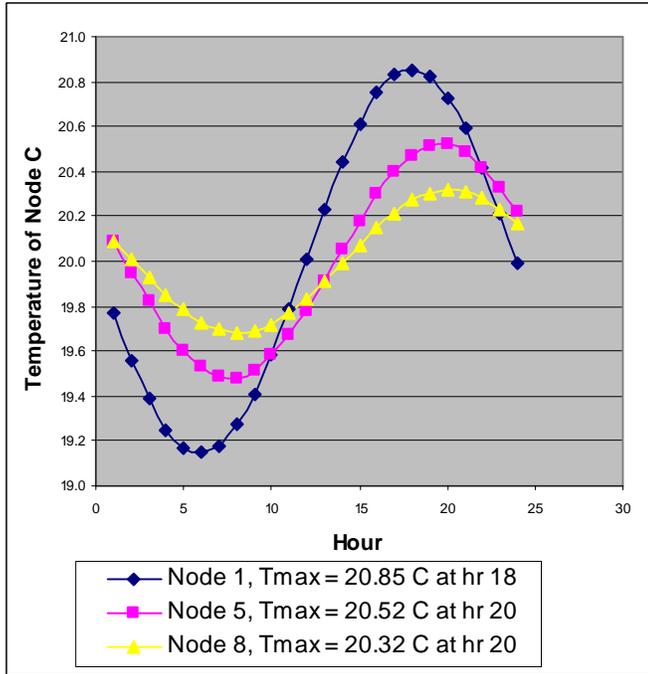


Fig. 3 24 Hour Temperature of Nodes 1, 5, and 8 with Thermal Mass = $1.4 \times 10^5 \text{ J/Km}^2$, $U_m = 5.1 \text{ W/m}^2\text{K}$, $t_{km} = 0.1 \text{ m}$ $T_{set} = 20 \text{ C}$, $T_m = 20 \text{ C}$, $T_{amp} = 10 \text{ C}$

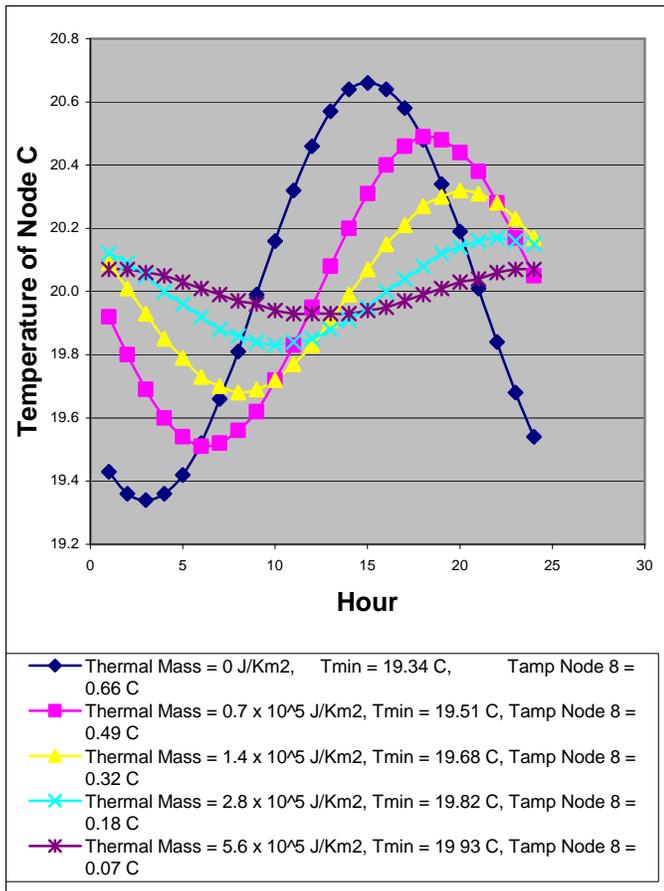
Equation 1 gives the total heat transferred between node 8 and the inside air over a 24 hour period. Solution of the equation for the zero-mass case when the amplitude of node 8 is 0.66 C gives a heat flux of 37.7 Whr/day. Solution of the equation for the mass case when the amplitude of node 8 is 0.32 C gives a heat flux of 18.3. The reduction in heat load is 52%, which confirms the finite difference solution.

$$\begin{aligned}
 Q \frac{Wh}{day} &= U \left\{ \left[T_{amp} \int_0^\pi \sin \theta d\theta \right] \right\} \frac{1}{2\pi} 24 \frac{hr}{day} & Eq.(1) \\
 &= 7.48 \frac{W}{m^2 \cdot K} 2T_{amp} rad \frac{1}{2\pi} 24 \frac{hr}{day} \\
 &= 37.7 \frac{Whr}{day} \text{ when } T_{amp} = 0.66 \text{ C and} \\
 &= 18.3 \frac{Whr}{day} \text{ when } T_{amp} = 0.32 \text{ C}
 \end{aligned}$$

U was determined by:

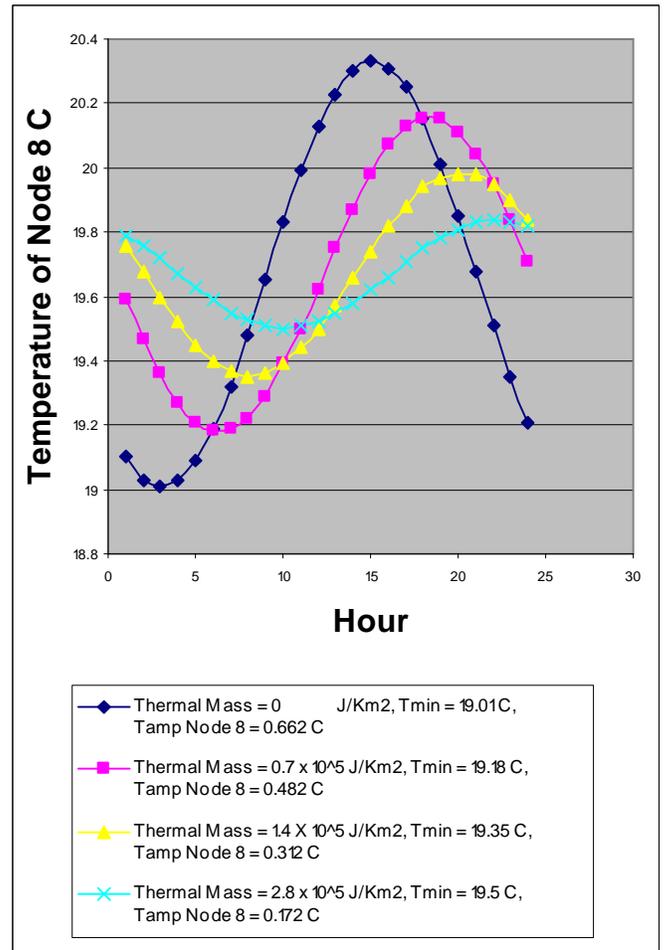
$$\begin{aligned}
 U &= \frac{1}{\frac{1}{h_i} + \frac{1}{U_{ii}}} \\
 &= \frac{1}{\frac{1}{8.26 \frac{W}{m^2 \cdot K}} + \frac{1}{79.5 \frac{W}{m^2 \cdot K}}} \\
 &= 7.48 \frac{W}{m^2 \cdot K}
 \end{aligned}$$

To further demonstrate the effects of thermal mass, Figure 4 plots the time-temperature profile for node 8 with increasing values of the thermal mass. For 0 thermal mass, the heating thermal load is 37.7 Wh/day; when the thermal mass is increased to $5.6 \times 10^5 \text{ J/K}\cdot\text{m}^2$, the heating thermal load is reduced to 4.3 Wh/day. The heating thermal load is defined as the quantity of heat that must be added to keep the inside air temperature constant. Similarly, the cooling thermal load is the quantity of cooling required to keep the inside air temperature constant. When the mean outdoor air temperature is the same as the indoor air temperature, as in this case, the heating and cooling thermal loads are equal. The time delay increases from 4 hours to 10 hours and the peak temperatures decrease from 0.66 C, to 0.07 C. Note that the thermal load, time delay, and temperature amplitude do not change linearly with respect to the amount of thermal mass.



**Fig. 4 24 Hour Temperature of Node 8 with Increasing Thermal Mass, T_{set} = 20 C, T_m = 20 C, T_{amp} = 10 C
T_{mean} of Node 8 = 20 C.**

The second case, with the mean exterior air temperature of 15 C, demonstrates the bounds or conditions under which thermal mass effects thermal load. In addition, it demonstrates the lower limit of thermal load below which increasing thermal mass has no effect on thermal load. Figure 5 plots the time-temperature profile of node 8 with increasing values of thermal mass. The mean exterior air temperature is 15 C. The exterior air temperature amplitude is 10 C. The figure shows that the temperature amplitude decreases as the value of the thermal mass increases, while the mean temperature remains the same. Thus, thermal mass affects only the amplitude, not the mean temperature.



**Fig. 5 24 Hour Temperature of Node 8 with Increasing Thermal Mass, T_{set} = 20 C, T_m = 15 C, T_{amp} = 10 C
T_{mean} Node 8 = 19.6685 C.**

The relation for the thermal load when the mean temperature of node 8 is below the interior air temperature, and the peak temperature of node 8 is above the interior air temperature is shown in Equation 2. This equation considers the difference between the interior air temperature and the node 8 temperature; hence, the appropriate U value is also between node 8 and the interior air. The equation is also valid if the exterior air mean temperature and amplitude were used with the overall U value of the wall. When the mean temperature of node 8 is set to 19.6685 C, Equation 2 gives the heating thermal load for zero thermal mass as 72.4 Wh/day . When the thermal mass is increased to $0.7 \times 10^5 \text{ J/Km}^2$, the heating thermal load for is 64.4 Wh/day . These results are equivalent to the thermal loads from the finite-difference simulation.

$$Q_{heat} \frac{Wh}{day} = U \times \left[\begin{array}{l} T_{set} - T_{mean \ node8} \\ \left[2T_{amp \ node8} \cos \left(\sin^{-1} \left(\frac{T_{set} - T_{mean \ node8}}{T_{amp \ node8}} \right) \right) \right] \\ + \left[-2(T_{set} - T_{mean \ node8}) \left(\frac{\pi}{2} - \sin^{-1} \left(\frac{T_{set} - T_{mean \ node8}}{T_{amp \ node8}} \right) \right) \right] \end{array} \right] \left\{ \frac{1}{2\pi} \right\} \times 24 \frac{hr}{day} \quad Eq.(2)$$

The relation for the thermal load when the mean and the peak temperatures of node 8 are below the interior air temperature is shown in Equation 3. The amplitude is reduced by additional thermal mass, but the summation of the temperature difference is the same as long as the peak node temperature is less than or equal to the interior air set point. In this case, the heating thermal load has a minimum value of 59.5 Wh/day.

$$Q_{heating} \frac{Wh}{day} = U \int_0^{24} (T_{set} - T_{node"8"}) dt \quad Eq.(3)$$

$$= U (T_{set} - T_{mean \ node"8"}) \frac{24h}{day}$$

MAXIMUM THERMAL LOAD REDUCTION WITH THERMAL MASS

Based on the results presented so far, the following observations about the relationship between thermal mass and heating load can be made:

a. When the mean cyclic exterior air temperature is equal to or greater than the interior set point temperature, the heating thermal load can be completely eliminated with thermal mass.

b. When the mean cyclic exterior air temperature is below the interior set point temperature, and the peak exterior air temperature is above the interior set point temperature, the heating thermal load can be reduced to a minimum of (variation of Equation 3):

$$Q_{heating} \frac{Wh}{day \cdot m^2} = U_{total} \frac{W}{m^2 C} (T_{set} - T_{mean \ exterior \ air}) C \frac{24hr}{day}$$

c. When the mean cyclic exterior air temperature and the peak exterior air temperature are below the interior set point temperature, thermal mass does not reduce the heating thermal load.

Similarly, the results presented so far support the following observations about the relationship between thermal mass and cooling load:

d. When the mean cyclic exterior air temperature is equal to or less than the interior set point temperature, the cooling thermal load can be completely eliminated with thermal mass.

e. When the mean cyclic exterior air temperature is above the interior set point temperature, and the minimum exterior air temperature is below the interior set point temperature, the cooling thermal load can be reduced to a minimum of (variation of Equation 3):

$$Q_{cooling} \frac{Wh}{day \cdot m^2} = U_{total} \frac{W}{m^2 C} (T_{mean \ exterior \ air} - T_{set}) C \frac{24hr}{day}$$

f. When the mean cyclic exterior air temperature and the peak exterior air temperature are above the interior set point temperature, thermal mass does not reduce the cooling thermal load.

USING WEATHER FILES TO DETERMINE MAXIMUM THERMAL LOAD REDUCTION WITH THERMAL MASS

Thermal mass has two effects on the temperature profile within the mass. First, thermal mass reduces the amplitude of the time-temperature profile. As the thermal mass approaches infinity, the amplitude approaches 0 and the temperature approaches the average of the temperature cycle. Thermal mass also delays the time-temperature profile. When the peak amplitude temperature is above the interior air temperature set point, decreasing the temperature amplitude will also reduce the thermal load. The average temperature difference between the exterior air temperature and the interior air temperature is reduced. However, when the peak exterior air temperature is below the interior air temperature, reducing the amplitude does not change the average temperature difference between the exterior air temperature and the interior air temperature.

To determine the minimum thermal load and the maximum thermal load reduction, it is only necessary to determine the average temperature of the exterior air temperature cycle and use this temperature difference between the exterior air temperature and the interior air temperature to calculate the thermal load for the duration of that cycle. Thus, Equation 4 shows the minimum thermal load obtained by increasing thermal mass as a function of exterior air temperature.

$$ThermalLoad_{minimum} = U \sum_{i=1}^n (T_{set} - cycle \ average \ temperature)_i (period \ of \ the \ cycle)_i \quad Eqn(4)$$

In many cases an accurate approximation to Equation 4 is obtained by using the 24 hour time average of the hourly solar temperature to calculate the thermal load (Equation 5).

$$ThermalLoad = U \sum_{i=1}^{8760} (T_{set} - \bar{T}_{24,i})_i$$

where

$$\bar{T}_{24,i} = \frac{\sum_{j=-12}^{12} T_{i+j}}{24} \quad Eqn(5)$$

To demonstrate the accuracy of Equation 4 and 5, a finite-difference program was created to model BESTEST qualification procedures for heavy and light buildings using the DRYCOLD weather file. No infiltration, internal generation, or windows were included in the finite difference program. The finite difference program showed a heating load reduction from 4.7 MWh/year to 4.24 MWh/year, which is a reduction of 10%. Applying Equation 4 with the DRYCOLD weather file resulted in a heating load reduction of 10%, whereas applying Equation 5 resulted in a heating load reduction of 9.5%. Similarly, the finite difference program predicted a reduction in cooling load from 1.11 MWh/year to 0.66 MWh/year, which is a reduction of 40%. Application of Equations 4 and 5 both resulted in a reduction of 40%. The Minneapolis weather file with the finite difference program predicted a heating load reduction of 7% and a cooling load reduction of 33%. Equation 4 predicts heating load reduction of 6% and a cooling load reduction of 31%. For Phoenix, the finite program predicted a heating load reduction of 38% and a cooling load reduction of 12%. Equation 4 predicts respectively 28.4% and 10.8%. The 9.2% difference in heating load reduction in Phoenix is attributed to estimating of the solar average on surfaces when applying Equation 4.

DESIGN EQUATION FOR THERMAL LOAD AS FUNCTION OF THERMAL MASS

Figure 6 plots the temperature amplitude of node 8 as a function of increasing thermal mass, when the mean exterior sol-air temperature is 20 C. Solution of the heat equation for a wall with no internal heat generation, suggests that this relationship has the form of (Equation 6):

$$Amplitude = k1 / \exp(k2 m cp) \quad Eq.(6)$$

where, k1 and k2 are constants, m is mass and cp is specific heat. For the conditions shown in Figure 5, a close agreement is obtained by the function

$$Amplitude = 0.66 / e^{(0.5 \times 10^{-5} \times mass)}$$

This function is also plotted in Figure 6, with close agreement between the two curves.

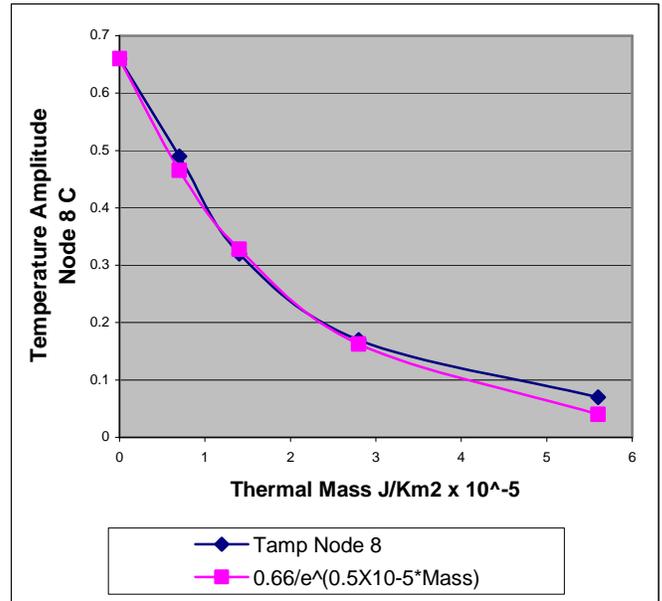


Fig. 6 Temperature Amplitude at Node 8 with Increasing Thermal Mass, Tset = 20 C, Tm = 20 C, Tamp = 10 C

Figure 7 plots the temperature amplitude of node 8 as a function of increasing thermal mass. The mean exterior sol-air temperature is 15 C. The function $0.66 / e^{(0.5 \times 10^{-5} \times mass)}$ is also plotted with increasing thermal mass. There is close agreement between the two curves. These results suggest that the decrease in the amplitude of the temperature variation within the thermal mass can be approximated by the function $1 / e^{(const \cdot thermal\ mass)}$.

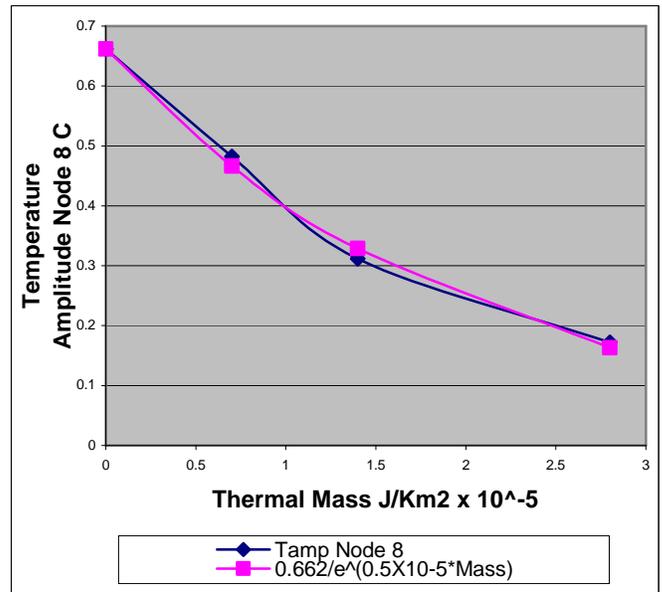


Fig. 7 Temperature Amplitude at Node 8 with Increasing Thermal Mass Tset = 20 C, Tm = 15 C, Tamp = 10 C

The location of the insulation relative to the thermal mass was also considered. To do so, the location of the insulation was switched from the outside to the inside of the wall. This change did not affect thermal load results.

The effect of the U-value of the thermal mass was then investigated by changing the thermal conductivity of the thermal mass while keeping the overall U-value of the wall the same. Figure 8 shows the change in amplitude with increasing thermal conductivity of the thermal mass. The amplitude of node 8, and therefore the thermal load, increases nearly linearly with thermal conductivity. It is noted, with 0 mass and with the overall U-value remaining constant, that the thermal conductivity has no effect on the amplitude of node 8 and therefore has no effect on the thermal load. Only as the thermal mass is increased does the increase in thermal conductivity result in an increase in thermal load. It is concluded that less thermal mass is required to obtain equal thermal load reduction if the thermal conductivity of the thermal mass is decreased. The thermal mass becomes more effective as its conductivity is decreased. This suggests that adding phase-change material to insulation increases the thermal mass effect.

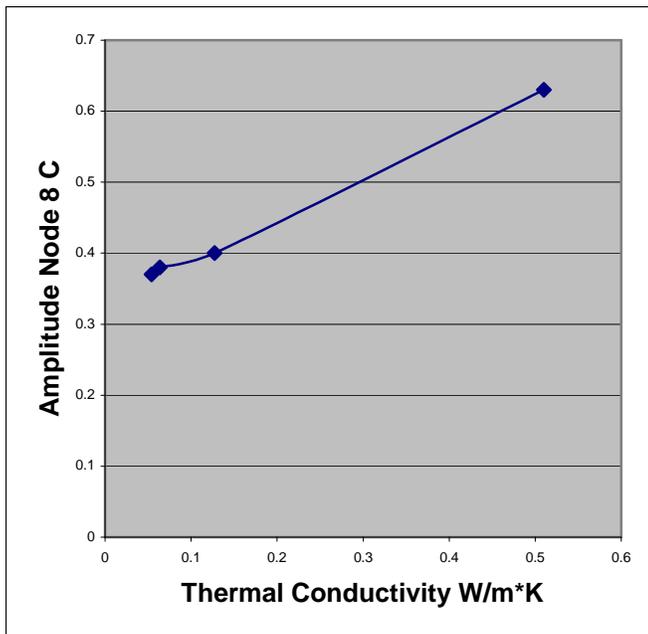


Fig. 8 Temperature Amplitude at Node 8 with Increasing Thermal Conductivity of Thermal Mass Tset = 20 C, Tm = 20 C, Tamp = 10 C

By extending Equation 2, the equation for thermal load as a function of thermal mass and thermal conductivity is given by Equation 7. Equation 7 is identical to Equation 2, but uses the functional approximation in Equation 6 for amplitude decay. Thus, Equation 7 is a closed-form approximation of simulation results.

$$Q_{heat} \frac{Wh}{day} = U \times \left[\begin{array}{l} T_{set} - T_{mean \text{ exterior air}} \\ 2T_{amp \text{ exterior air}} \cos \left(\sin^{-1} \left(\frac{T_{set} - T_{mean \text{ exterior air}}}{T_{amp \text{ decay}}} \right) \right) \\ -2(T_{set} - T_{mean \text{ exterior air}}) \left(\frac{\pi}{2} - \sin^{-1} \left(\frac{T_{set} - T_{mean \text{ exterior air}}}{T_{amp \text{ decay}}} \right) \right) \end{array} \right] \frac{1}{2\pi} \times 24 \frac{hr}{day}$$

$$T_{amp \text{ decay}} = \frac{T_{amp \text{ exterior air}}}{e^{\left(\frac{0.22C\rho_m\rho_m k_m \omega}{k_m^{0.333}} \right)}}$$

If $T_{amp \text{ decay}} + T_{mean \text{ exterior air}} < T_{set}$, then $T_{amp \text{ decay}} = T_{set} - T_{mean \text{ exterior air}}$ Eqn(7)

Figures 9, 10, 11, and 12 compare the results from the closed-form approximation given by Equation 7 with the finite-difference simulation. In all cases, Equation 7 is in close agreement with the finite difference solutions. Thus, Equation 7 can serve as a guide for sizing thermal mass according to weather conditions.

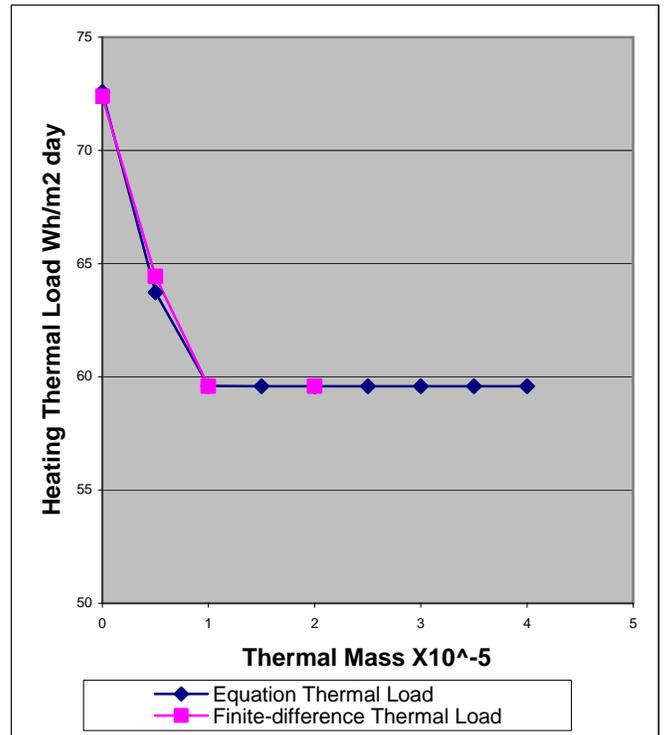


Fig. 9 Equation Thermal Load with Finite-Difference Thermal Load Tset = 20 C, Tm = 15 C, Tamp = 10 C, km =0.52 W/m*K

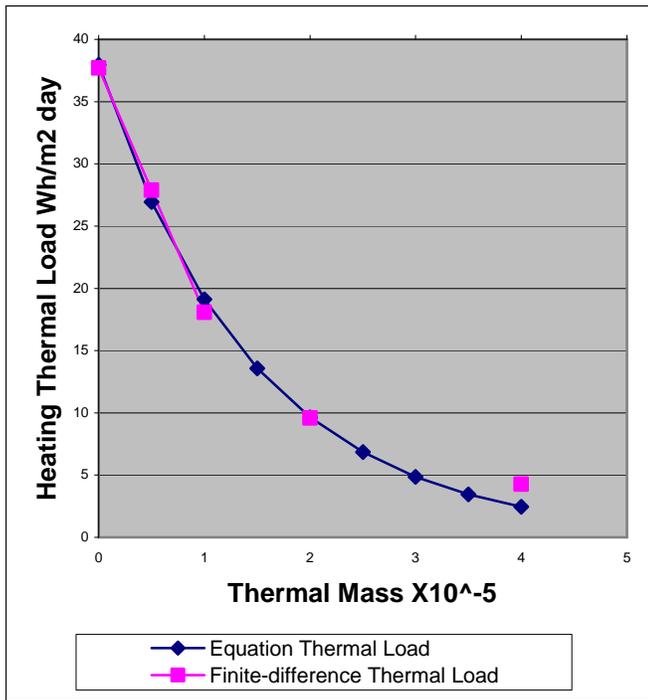


Fig. 10 Equation Thermal Load with Finite-Difference Thermal Load, $T_{set} = 20\text{ C}$, $T_m = 20\text{ C}$, $T_{amp} = 10\text{ C}$, $k_m = 0.52\text{ W/m}^2\text{K}$

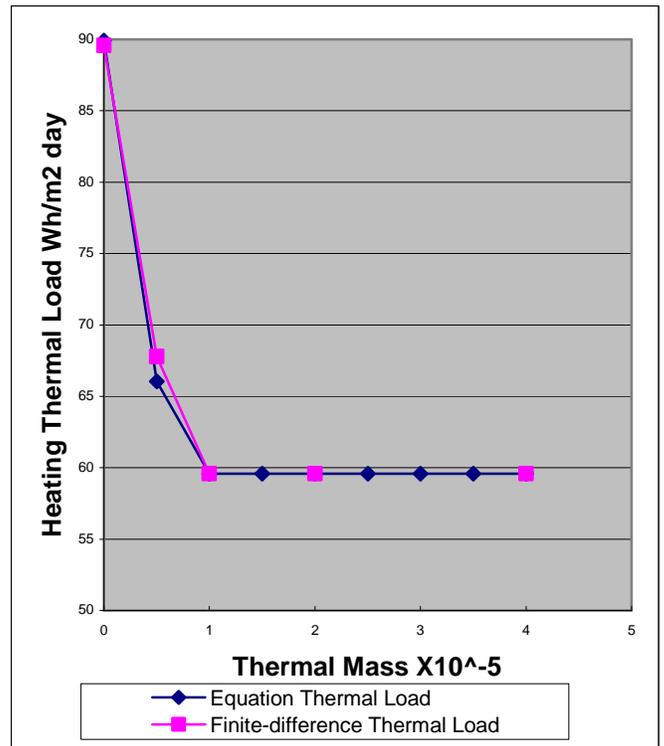


Fig. 12 Equation Thermal Load with Finite-Difference Thermal Load $T_{set} = 20\text{ C}$, $T_m = 15\text{ C}$, $T_{amp} = 15\text{ C}$, $k_m = 0.0638\text{ W/m}^2\text{K}$

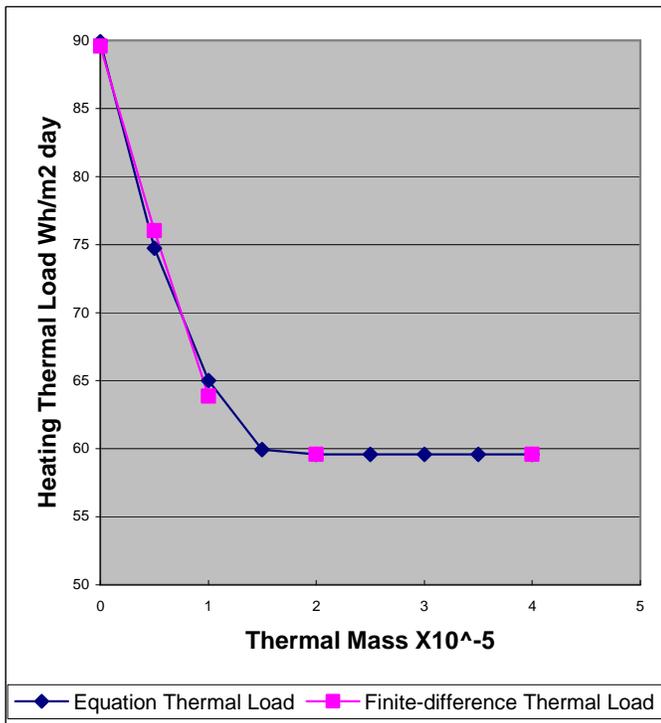


Fig. 11 Equation Thermal Load with Finite-Difference Thermal Load $T_{set} = 20\text{ C}$, $T_m = 15\text{ C}$, $T_{amp} = 15\text{ C}$, $k_m = 0.52\text{ W/m}^2\text{K}$

SUMMARY AND CONCLUSIONS

Thermal mass reduces the magnitude of diurnal temperature variation in walls and building structures. If the peak cyclic exterior air temperature is higher than the interior air temperature, adding thermal mass reduces the thermal heating load. If the peak cyclic exterior air temperature is lower than the interior air temperature, adding thermal mass will not reduce the thermal heating load. Similarly, increasing the thermal mass reduces the cooling thermal load only when the minimum exterior air temperature is below the interior air temperature set point.

Equation 4 can predict the minimum thermal load that can be obtained with thermal mass. In many cases, Equation 5 can approximate the minimum thermal load with good accuracy. In all cases, Equation 5 will yield conservative results with a higher thermal load than the true minimum. In addition, the reduction in temperature amplitude, and therefore the peak thermal load, can be closely approximated by Equation 6. Equation 7 can be used as a design guideline for calculating thermal load as a function of thermal mass and thermal conductivity. To apply Equation 7 a peak amplitude temperature is required. The heating and cooling requirements are determined separately. With the cooling requirements as an example, the greatest seasonal peak minimum temperature is plotted against increasing thermal mass. As this is an exponential function, the slope quickly approaches zero and defines a quantity for the thermal mass. For verification, the average peak and minimum temperatures can also be plotted against the thermal mass.

Taken together, these equations make it possible to consider how thermal mass will affect building loads without detailed and time consuming simulation.

This model considers a wall with 2 layers of insulation and a single layer of thermal mass, in which the thermal mass of the insulation layers is considered negligible. We believe that this model is generally applicable to many common types of wall construction, but not necessarily to all types. Thus, the results pertain only to walls which can be approximated using this model.

The exterior air temperature is used as simulated by the sinusoidal function. The authors believe the final results will be more accurate if the ambient driver is represented by a sinusoidal approximation of sol-air temperature rather than the exterior air temperature. The sol-air includes the solar radiation as:

$$sol - air = T_{air} + I/h.$$

where:

$$I = \text{solar radiation}$$

and $h = \text{convection coefficient on exterior surface.}$

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